THE USE OF LASER AND X-RAY SCANNING TO CREATE A MODEL OF THE HISTORIC KOH-I-NOOR DIAMOND

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For centuries, the Koh-i-Noor was one of the world's largest diamonds. It was recut in 1852 from its historic form of 186.1 old carats to an oval of 105.60 modern carats. While the modern shape is well documented, the original form is not. Fortunately, the Natural History Museum in London commissioned a mold of the diamond in 1851 before it was recut. Using one of two plaster replicas made from this mold, the present study captured the surface topology of the original Koh-i-Noor through photography plus laser and X-ray scanning methods. A crystallographic analysis of the major facets determined the diamond's orientation within a "perfect" diamond crystal, which was used to refute one theory about the diamond's genealogy. Computer modeling established the orientation of the recut diamond within the historic version. Information from this study was used to create an accurate replica from cubic zirconia.

The Koh-i-Noor is one of the great historic diamonds (figure 1), a gem that has figured prominently in south Asian history for hundreds of years. The precise history of the Koh-i-Noor prior to the mid-1500s is in doubt (Mawe, 1815; Balfour, 2000). Some sources claim the stone was discovered in India 5,000 years ago, while others place it in the 1300s or 1500s. It was owned by a long succession of rulers and warlords over the ensuing centuries, finally ending up as part of the English Crown Jewels after the British conquest of the Punjab. Queen Victoria had the stone recut to its current form in 1852 (see box A for more information on the diamond's history).

Although much is known about the original Koh-i-Noor, drawings of its form are scarce and contradictory (figure 2). As a result, there is insufficient historical data to recreate the diamond with any accuracy.

However, the Natural History Museum in London created a plaster mold of the diamond in 1851, prior to the recutting, and from this two plaster casts were made. The discovery of a firebrick mold in December 2007 suggests that at least one lead model was also made, but there is no documentation to confirm this. The two plaster casts are the only replicas known to have been made from the original stone (a cubic zirconia [CZ] replica has been cut based on one of these plaster forms, as described in Hatleberg, 2006, but no details as to the specific techniques used were provided in that report). One of the plaster casts is on permanent display at the museum, but the other was analyzed for this study using a combination of photographic techniques and laser and X-ray scanning technologies to create a computer model of the original Koh-i-Noor. This model was used to determine the alignment of the historic stone within the original diamond crystal (and test one theory of its origin), analyze the

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Figure 1. The Koh-i-Noor, currently the centerpiece of the Queen Mother's crown in the English Crown Jewels, is perhaps the most famous diamond in the world. It was recut to its current size of 105.60 ct in 1852. A replica of its historic form (inset) was prepared as part of this article. Crown ©/ The Royal Collection © Her Majesty Queen Elizabeth II; inset photo by S. Sucher.

recutting of the stone, and create a CZ replica to evaluate historical reports of its optical performance.

MODELING BACKGROUND

Famous diamonds have been replicated from glass, crystal, lead, and plaster for hundreds of years. More recently, various computer-aided techniques have been added to the modeler's tools (e.g., Attaway, 2005). Some, such as photography, photogrammetry, and photomodeling, rely on photos to reconstruct the stone. Laser and X-ray scanners, as well as optical scanners, can be used with either the original stone or a model to scan the surface topography and generate reflections from which a digital three-dimensional (3-D) model of the original can be created.

The study described here combined photography with laser and X-ray scanning. Initially, only the photographic process was employed. This consisted of photographing the plaster model from many angles and using these photos to generate line drawings of the facet pattern and other surface features. The line drawings served as design templates, on the basis of

BOX A: THE STORY OF THE KOH-I-NOOR

Although it is neither the largest nor the most dazzling diamond ever found, the Koh-i-Noor's fame and colorful background are unsurpassed. The stone's origins and early history have never been conclusively established—apocryphal accounts have placed its discovery as early as 3000 BC. It is widely believed to have come from India's Golconda region, an early source of high-quality colorless diamonds. The first reliable reference to the Koh-i-Noor is believed to be in the memoirs (circa 1526) of Babur, founder of the Mogul dynasty, who wrote of the Sultan of Delhi capturing the then-unnamed diamond in 1304 (Streeter, 1882). Babur himself had captured the stone in 1526 after defeating its previous possessor, the Rajah of Gwalior (Balfour, 2000).

For centuries, the original 186 ct (191.03 modern carats) diamond changed hands repeatedly and often by violence, a coveted spoil of the wars between the Mogul empire, India, Afghanistan, and Persia. Its name, said to have been exclaimed by the Persian Emperor Nadir Shah upon seeing the gem for the first time after the sack of Delhi in 1739, is Persian for "Mountain of Light." The British East India Company acquired the Koh-i-Noor in 1849 with the British conquest of the Punjab. In 1850, it was presented to Queen Victoria at a ceremony marking the 250th anniversary of the founding of the East India Company (Copeland, 1974), and it has remained in British possession ever since.

The Koh-i-Noor first went on public display at the Great Exhibition, held in London's Hyde Park from May to October of 1851. The diamond was displayed in a large glass case and illuminated by either candles or gas lamps, hardly suitable conditions for brilliance or sparkle. *The Times* reported that while the gem drew the exhibition's largest crowds, many were disappointed by its rather ordinary appearance (Balfour, 2000). The following year, Prince Albert gave it to the crown jewelers, Garrard & Co., for recutting (Blair et al., 1998). Garrard's brought several master cutters from Coster Diamonds in Amsterdam to London to oversee the job (figure A-1), and 38 days later the Koh-i-Noor had been trimmed to 105.60 (modern) ct, just over half the original size. The resulting oval-cut diamond was set in a tiara for the queen.

Opinions of the recutting were mixed. Prince Albert, who had been closely involved with the operation, was openly dissatisfied with the outcome, especially the great loss of weight it had entailed (Streeter, 1882). There is also a perhaps-apocryphal story that when the Sikh Prince Dhulip Singh, the Koh-i-Noor's last non-British owner, was shown the recut diamond by Queen Victoria, he was rendered speechless for some minutes afterward (Blair et al., 1998). That the recutting was unnecessarily rushed seems clear, judging by the number of correction facets on the stone (Blair et al., 1998).

Gemologically, the Koh-i-Noor is a D-color type IIa stone (Blair et al., 1998). Unlike similar classic "Golconda diamonds" that show pinkish orange fluorescence (Fritsch, 1998), the Koh-i-Noor is inert to long-wave UV radiation but shows a weak green reaction to short-wave UV. Although it has not been given a clarity grade, it shows several feathers/gletzes and a black inclusion under its table (Blair et al., 1998).

Since the Koh-i-Noor's acquisition by the British,

which the diamond could be virtually "cut" using a software program designed for three-dimensional gem modeling. The data from this computer model were then used to create a CZ replica that more closely mimicked the diamond than the materials historically used.

The complexity of the plaster model's surface topography made it extremely difficult to resolve the Koh-i-Noor's features photographically. Although photos alone can be used to produce a computer-generated and physical replica of any original, any assumptions as to facet settings based on the photos have a high probability of causing errors. These can be minimized through extensive iterations (i.e., "trial and error") of the resultant computer model—making adjustments to each facet until the computer model matches the photos as best as possible—but this is extremely time consuming for a stone of such complexity. It became apparent that a different technique was needed to improve accuracy, as this would be our only opportunity to have unfettered access to the 150+-year-old plaster replica. The technique had to be objective and generate highly accurate data that precisely captured the diamond's surface topology. Laser and X-ray scanning methods meet both these criteria.

METHODS AND RESULTS

The Photographic Method. We applied the photographic method to capture the outline of the plaster replica and every surface detail, including the pattern,

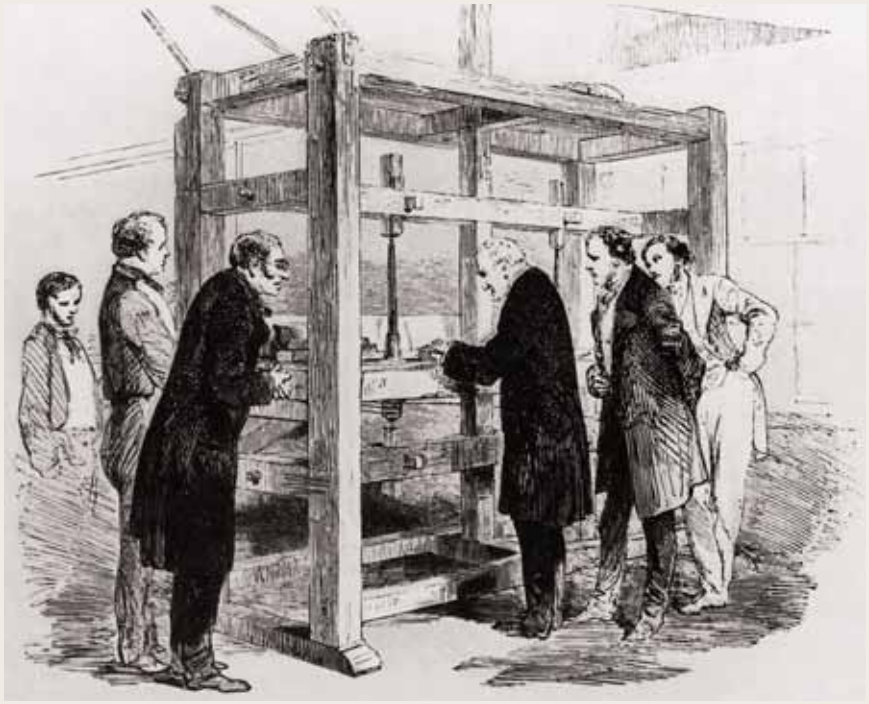


Figure A-1. This drawing from The Illustrated London News *(July 1852) depicts the recutting of the Koh-i-Noor by a team from Garrard & Co., the Crown Jewelers, and Coster Diamonds of Amsterdam.*

only female members of the royal family have worn it. This may be because the diamond is said to carry a curse that brings its male possessors great misfortune—though the Indian legend is the exact opposite, reflecting the many powerful men who possessed it (Balfour, 2000).

Today, the Koh-i-Noor resides in the Tower of London, where it is the centerpiece of the Queen Mother's 1937 coronation crown (again, see figure 1), seen by millions of visitors each year. Yet the diamond's future is still not entirely settled, as the governments of India, Pakistan, Iran, and Afghanistan have all claimed ownership of it and demanded its return at various points over the past few decades. Controversy flared up again in 2002, after the Queen Mother's funeral. Her crown, including the Koh-i-Noor, was placed atop her casket during the ceremony, a gesture that angered Indian politicians, who considered it an unnecessary flaunting of imperial plunder ("Diamond teaser. . .," 2002).

Even as the legendary diamond enjoys a seemingly quiet "retirement," its notoriety continues to grow.

shape, and size of the facets, as well as four surface cavities that held the original diamond in its prong setting (Tennant, 1852, p. 82). Using a Canon Powershot A40 digital camera with 2-megapixel resolution, we took 10 series of photos from different perspectives—top, bottom, and side views at every 45° of rotation—to create the line drawings (e.g., figure 3).

Many facets on the plaster replica were difficult to discern in the photos. Plaster is a soft material that erodes over time. Facet edges become rounded, often making the outlines difficult to see. In addition, the angular difference between adjacent facets was minimal, in many cases less than 3°, causing one facet to "blend" into its neighbors. This was resolved by taking many photos under low-illumination, low-angle conditions. A 40 watt incandescent light source was rotated about the model in a darkened room, causing facet junctions to alternately appear and disappear (figure 4, photos). Over 700 pictures were taken in various lighting conditions.

Next, we created the line drawings by importing a baseline photo into Adobe Illustrator 9.0 and tracing all visible surface details. Then we imported another photo from that series, with the light in a different position, and the newly visible features were added to the line drawing. Additional photos were imported until all features visible with the changing light positions had been incorporated and the resulting line drawing was complete (see, e.g., figure 4, bottom).

The line drawings captured the facet details, stone outline, and all other features necessary to describe the surface topology of the plaster replica.

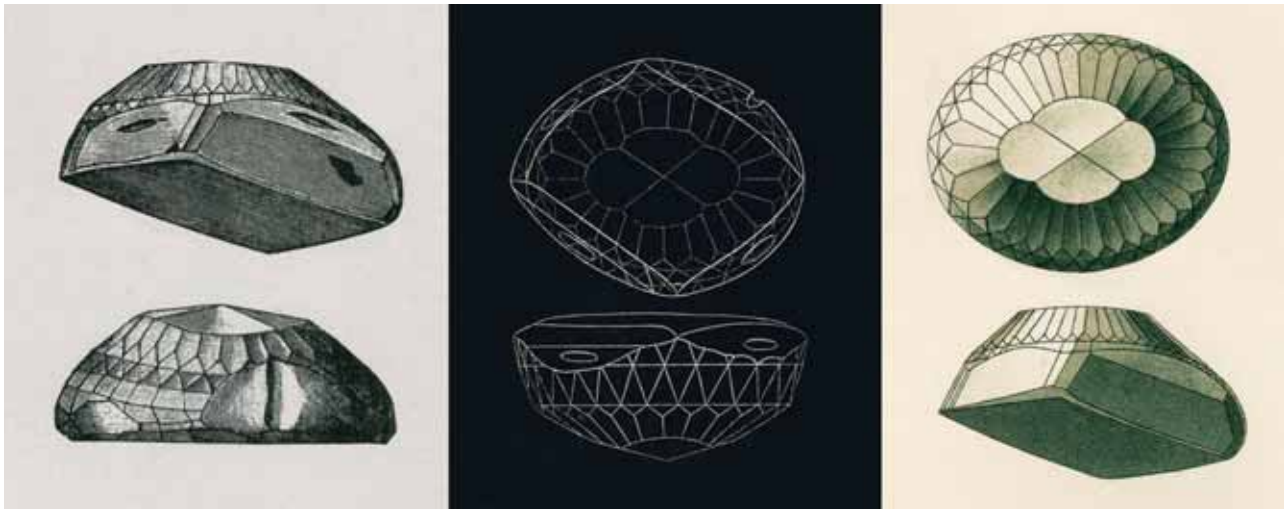


Figure 2. These historic drawings of the original Koh-i-Noor diamond show conflicting appearances: left—Tennant (1852, p. 83); center—Dieulafait (1874, p. 95); and right—Bauer (1968, plate X).

As noted earlier, though, relying on photography alone to generate line drawings introduces additional sources of error. These were addressed as follows:

Viewing Angle. Photos could not be taken randomly; they had to be shot from specific viewing angles that were accurately reproducible. The solution was a compass rose with graduated markings like those of the index wheel on a faceting machine. The plaster replica was placed on this compass rose and held stationary (figure 5). Photos were taken from different angles by rotating the compass rose around a fixed axis and maintaining the camera in a fixed position.

Parallax. Parallax (the apparent displacement of a feature as seen from two different points) was especially acute for facets visible on the far left or right in any photo, due to the diamond's pronounced curvature. This was mitigated by giving priority to those facets in the center of a photo whenever there was a discrepancy between adjacent views.

Perspective. All photos had to be taken orthogonal to a diamond axis. Side views were directly from the side (again, see figure 4), with the camera and replica positions kept fixed. Any angling up or down would distort facet shapes.

Depth of Field. All photos had to be in focus. An out-of-focus picture causes a slight displacement of surface features when the line drawing is generated. This effect is especially acute on sharply curved surfaces such as those of the Koh-i-Noor, which became apparent during computer modeling. A facet visible in one photo would be slightly displaced on the computer model in an adjacent view. The center facets in the picture would be in focus, but those on either side would be blurred and their

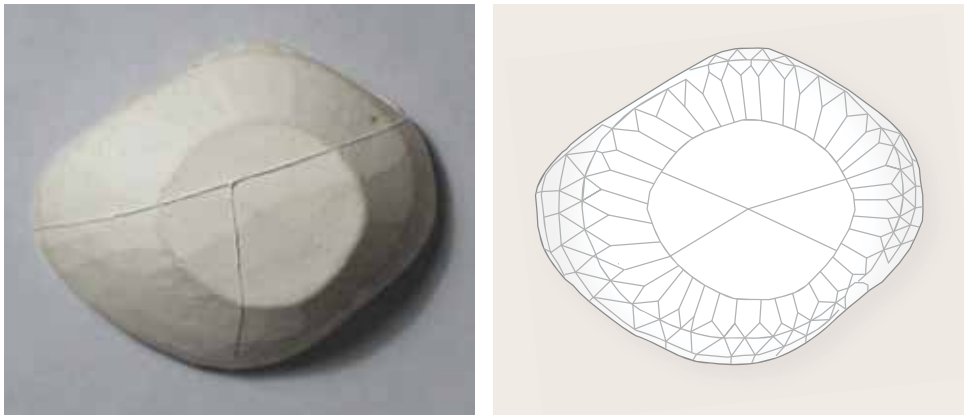


Figure 3. Photos of the plaster model (left) served as a guide to generate line-drawing templates, from which facet pattern and other surface features could be derived (right). Not all facets could be discerned from the photos. Photo by S. Sucher.

apparent positions in the photo slightly different from their real positions. Again, priority was given to facets in the center of any photo.

Line Drawings. Creating a line drawing that captures diamond outline, facet junctions, and other features is highly subjective when these features are not well defined. Misplacing the endpoint of a facet junction by even one pixel has a cumulative effect in introduced error. This was mitigated by expanding the photos approximately 5× in Illustrator to trace the facet junctions. The resultant drawing was exported as a .wmf (Windows Metafile Format) file, as this vector-based format allows expansion and contraction of the line art without the associated ragged lines of raster-based drawings. The wmf drawing was used as the overlay graphic during computer modeling.

Indistinct Facets. Determining facet shape and size from the photos was very difficult because each facet tended to blend in with adjacent ones, as described above. Low-angle lighting (again, see figure 4) mitigated most of this effect. Approximately 10 facets were so indistinct as to be nonexistent, but they could be inferred. A study of ancient Indian cutting techniques (Meen, 1968) revealed several historic diamonds in the Iranian Crown Jewels that were cut similarly to the Koh-i-Noor. All have four main crown facets surrounded by long facets, which in turn were bounded by very small triangular facets. The succeeding rows consist of small triangular or rectangular facets. On the Koh-i-Noor, most of the "missing" facets were the long facets and the very small triangular ones adjacent to them. However, the row of facets below these was sufficiently defined to create the boundaries of the missing facets by emulating the pattern found on the other diamonds (figure 6).

The Scanning Process. Although with the photos we were able to successfully capture the surface topography of the plaster cast, the limitations of the photographic method required a more accurate method of capturing data for computer modeling, so we turned to commercially available laser and X-ray scanners. The Helium 1:4 digital laser scanner by OctoNus Software Ltd. offered the necessary precision and was able to accommodate an object roughly 41 × 31 × 17 mm. A sister company, SkyScan, manufactures a high-resolution X-ray scanner (model 1172) with similar capabilities. The resulting scan data could be interpreted using a proprietary software program, Diamond Calculator 3.0 (also referred to as DiamCalc 3.0), which gives the angle and index of each planar surface generated by the scan data. (The *angle setting* is the placement of the cutting angle in relation to a horizontal reference; the *index setting* refers to the degree of rotation around a central axis.)

The plaster replica was scanned on both the laser and X-ray machines in Belgium. Afterward, the replica was mailed directly back to the museum in London and the data were forwarded to the authors for analysis.

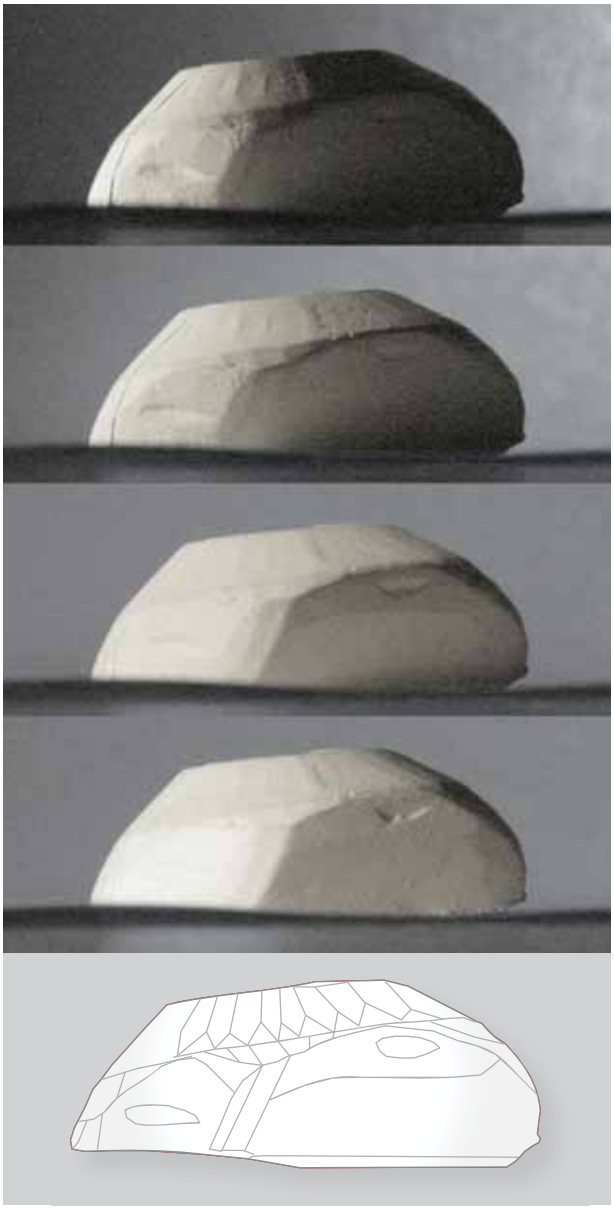


Figure 4. The four photos shown here produced this line drawing of the original Koh-i-Noor. The oval features are two of the cavities in the surface that were used for the prongs in the original mounting. Photos by S. Sucher.



Figure 5. The compass rose and model demonstrate how rotational alignment was maintained. The camera remained fixed as the diamond and compass rose were rotated. Photo by S. Sucher.

The Helium laser scanner has the following specifications: laser—635 nm; field of view—43.3 × 56.8 mm; rough capacity—5–500 ct; diamond holding—vacuum/glue assisted; 3-D model accuracy—better than 28 µm; a 1:4 lens; and CCD camera resolution—1920 × 1080 pixels.

The specifications of the X-ray scanner are: X-ray source—20–100 kV, 10 W, <5 µm spot size; X-ray detector—10 megapixel or 1.3 megapixel cooled CCD fiber-optically coupled to scintillator; detail detectability—<1 µm with 10 megapixel camera, <2 µm with 1.3 megapixel camera; size capacity—68 mm in diameter with 10 megapixel camera; reconstruction—single PC or cluster volumetric reconstruction (Feldkamp algorithm).

Although we used both tools to generate data, only the laser scan data were extensively analyzed once it was determined that the X-ray data closely matched them. The laser scan data generated 5,047 planar surfaces that defined the model's topography. Those planar surfaces were then reduced to the 169 facets shown on the line drawings. Thus, an average of 30 planes (5,047÷169) defined any given facet.

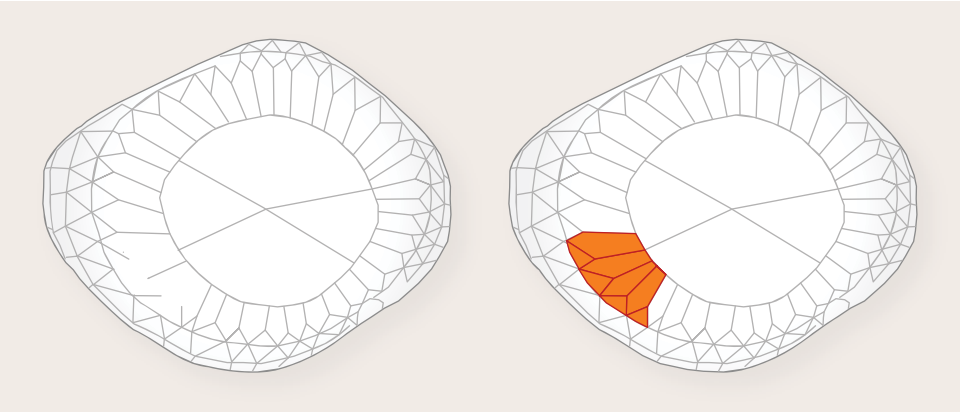


Figure 6. Some "missing" facets (left) could not be discerned in the photographs, but they were known to exist from analysis of the surrounding facets. The missing facets were filled in (right) by matching patterns on historic diamonds in the Crown Jewels of Iran with cuts similar to that of the Koh-i-Noor.

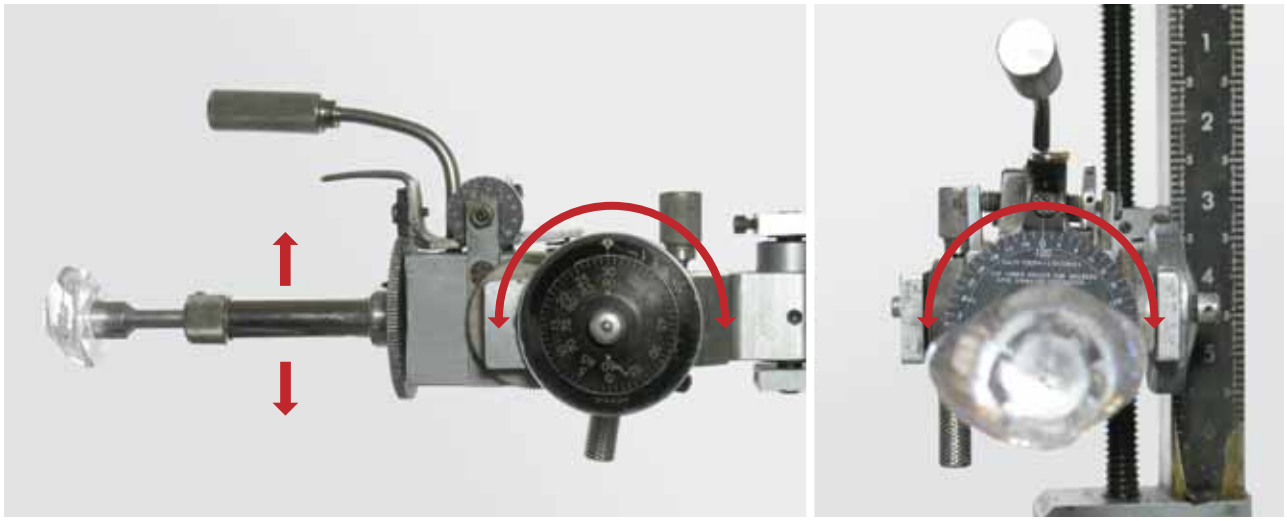


Figure 7. Each facet has a unique angle and index setting, as placement of the CZ model on this faceting machine illustrates. On the left: As the arm moves up and down, the angle dial rotates and displays the angle (0 reference is vertical). The arm position can be fixed at any angle setting to 0.25° accuracy. On the right: The arm is rotated about its longitudinal axis. Teeth on the gear behind the diamond allow any index setting to be fixed. A smaller wheel (the "index vernier" or "cheater," shown above and to the left of the index dial in the image on the left) allows finer settings to less than 0.10° accuracy. Photos by S. Sucher.

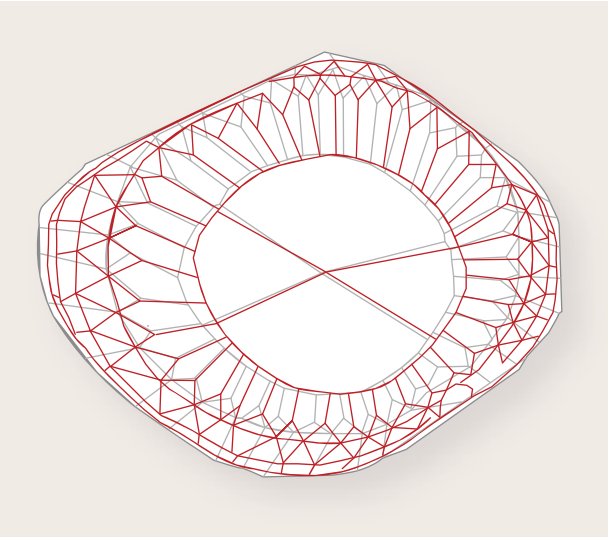
Computer Modeling. The line drawings created from the photos were used as design templates to virtually cut the diamond on the computer using GemCad, a software program for 3-D gem modeling. As noted above, the photographic process is a highly iterative one, where facet settings are derived primarily through trial and error. Because the original Koh-i-Noor was such a complex stone, we felt that placing the four large facets on the table in the computer model was a necessary first step in cutting (to both simplify the process and reduce the number of iterations needed). The appropriate line drawings were analyzed in Illustrator to determine the approximate settings needed to create these facets.

Cutting a facet, whether virtual or actual, requires the identification of two variables: the angle setting and the index setting (figure 7). These two settings are difficult to establish using the photographic technique, as they are determined through trial and error during computer modeling. Side photos showing the four crown facets were analyzed to determine the angle settings. Index settings were estimated using stonecutter judgment. These angle and index settings were entered in the modeling software to "cut" each virtual facet, and then they were iterated until facet size, shape, location, and intersections with neighboring facets matched the line drawing (figure 8). Dozens of iterations were required for just the first four facets, as the low crown angles had multiple index-setting solutions.

Once these four facets were placed on the model, the settings for adjacent facets were determined by resolving the intersections. Again, this process was highly iterative.

A limitation of both DiamCalc and GemCad is that they cannot model depressions such as the four cavities visible in the photos. To duplicate

Figure 8. This overlay of the modeling workspace with a preliminary computer model (gray) and line drawing (red) shows how real-time feedback is provided to ensure a virtual facet matches its counterpart in the line drawing. If the facet is not correct, its black outline will not match the red template.



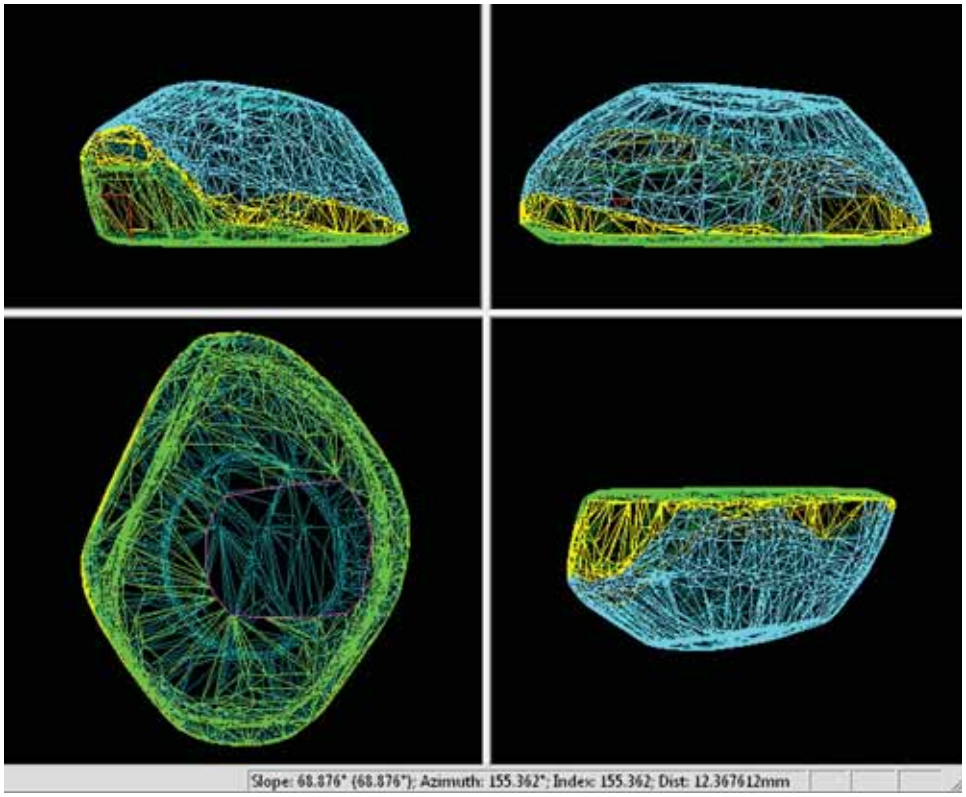


Figure 9. This graphical depiction of the scan data shows how the scan model can be rotated in all directions. Note the large triangular facet outlined in red in the upper left view, selected with a computer mouse. The angle (slope) and index (azimuth) of that facet (the facet settings) are displayed at the bottom of the screen. The distance value is the center-to-facet distance and was not used.

these cavities on the CZ replica, we used 10× magnification and a table gauge accurate to 0.1 mm to physically measure their dimensions during the cutting process.

The virtual solid generated by the scan data was displayed graphically in three dimensions with full rotation on the X, Y, and Z axes (figure 9). The settings for any particular plane could be found by positioning the cursor over that plane in the scan file. A readout provided facet settings to 0.001 in angle and index.

For the computer model, we overlaid each line drawing on the computer workspace and the scan picture (e.g., figure 10). We computed settings for each facet using the following procedure:

1. The facet of interest was identified in the line drawing.
2. The scan data were brought to the top, and the average of all facet settings in that area was determined.

Figure 10. In this overlay of a line drawing (red) on the scan data, the outlined yellow facet is the facet to be "cut" in GemCad. It shows the planes from the scan data that define that facet.

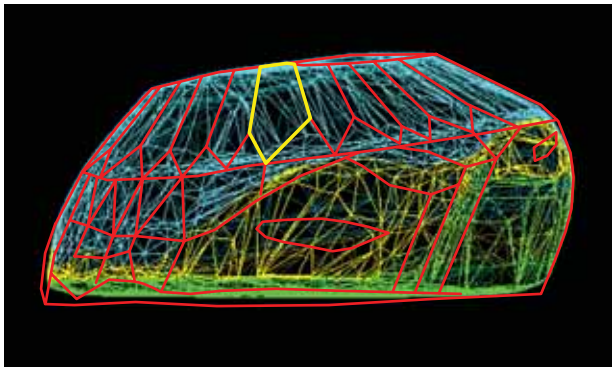


TABLE 1. Angles and indices of the planes outlined by the highlighted facet in figure 10.[a]

Angle	Index[b]
40.34°	227.91
41.83°	229.14
42.16°	230.04
42.31°	230.34
42.46°	230.92
43.82°	234.58
44.96°	238.03

[a] *Only planes 2–5 (highlighted here) were used to determine an average of facet settings, as the others have significantly different values.*
[b] *360 index wheel.*

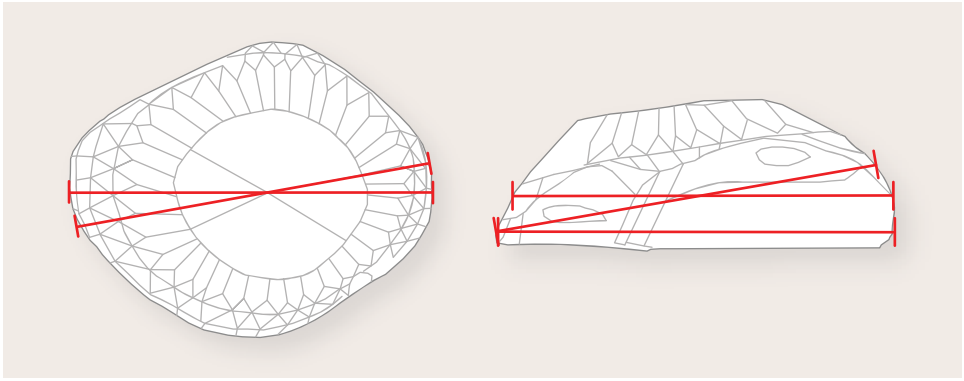


Figure 11. Sources of error that lead to inconsistent caliper measurements: (left) lack of girdle facets, combined with the ever-changing outline, makes reproducible positioning of calipers problematic (lines indicate axes of measurement); and (right) lack of a defined girdle makes it difficult to consistently position calipers so that they remain parallel to the model's base.

3. The workspace was placed on top, and the average of the settings was used to "cut" that facet in the computer model.

4. The new facet was compared to the line drawing to ensure it had the proper shape and size, and all facet intersections matched the drawing. If all three criteria were not met, then angle, index, and/or depth of cut were successively iterated within the range of angle- and index-setting values dictated by the scan data until the modeled facet matched the line drawing.

As an example of this process, table 1 lists the angles and indices of the larger planes for the facet described in figure 10. These were iterated in GemCad within the upper and lower values here until this facet matched the shape and size dictated by the line drawing.

Thus, the planar surfaces were reduced to the required 169 facets by using the line drawings to define the facet pattern and DiamCalc 3.0 to determine the angle and index settings of each facet from the laser scan data; both were then used to generate a computer model with GemCad.

The shape of the final GemCad model was then compared to the DiamCalc model from multiple perspectives to ensure the outlines matched. Any discrepancies were noted, and these areas were virtually "recut" until the model matched the scan data. At that point, the computer model was considered accurate.

This computer model could not be used directly to create a physical replica, as the model is dimensionless. The data alone could be used to cut a stone of 5 ct or 500 ct. GemCad has an algorithm for generating stone width, and hence length and depth, but any errors or assumptions during modeling would affect the result. Calipers could not be used to measure the plaster replica because its unusual shape and sidewall angles precluded reproducible measurement (figure 11). Values were in the vicinity of 41.7 × 33.9 × 16.7 mm, but they were highly variable depending on where the calipers were placed and whether the plane of measurement was parallel to the diamond's base. Even with utmost care, any measurement series had values that varied as much as 0.5 mm in length and width.

Replica dimensions were determined by exporting the .dmc format of the scan model as a .dxf file so it could be imported into TurboCad Pro 10 for measurement. The maximum dimension was determined by rotating the scan model one degree at a time until the longest measurement was achieved. Width was measured orthogonally to the length axis, and depth was provided by the scanning software. This resulted in dimensions of:

Length = 41.78 mm
Width = 33.63 mm
Depth = 16.56 mm

The dimensions were further adjusted after analysis revealed that these values caused a discrepancy with the historical Koh-i-noor's reported weight (see Weight Analysis below).

Physical Modeling. The CZ replica was created with the cutting instructions generated from the GemCad data. These instructions contained the angle and index setting of each facet, plus actual-size and magnified views from each viewing angle, and various templates for facet placement.

Two related factors that had to be considered during the computer and physical modeling processes were stone center and dop center. Stone center was critical for computer modeling, as facets are placed as a function of distance from this point. Since the computer data were being used to cut a physical replica, the axis of rotation during computer modeling (stone center) had to be aligned with the axis of

rotation on which the CZ replica was mounted during faceting (dop center, see figure 7, right).

Stone center (0,0) during computer modeling was achieved by creating a template of the outline from the top-down view and centering the virtual workspace in the X and Y directions within this outline. The computer model was then created around this 0,0 reference. Once the computer model was complete, a set of crosshairs was superimposed and centered in both the X and Y directions on the outline template. The template was then printed at actual size and glued to the rough CZ. A jig and specially built dop were used to align the dop axis with the crosshairs on the template on the rough at 10×. Stone center was now aligned with dop center, and cutting could begin.

The four largest facets were cut first using the angle and index settings generated by GemCad. Computer modeling showed that they had to be cut in a specific order to a specific depth. The depth issue was resolved by using templates and measurements to initially place these four facets in their approximate positions. Final placement did not occur until the first surrounding row of long facets was in place, whereupon another template was used to ensure each facet was the appropriate size, and the length of the facet junctions around these four facets were exactly those dictated by computer modeling (using a table gauge and 10× magnification).

The remaining facets were placed using the settings derived from computer modeling. The depth of cut (distance from stone center to facet), though readily available from the modeling data, could not be used during actual cutting, as there was no way to measure it.

However, this limitation could be overcome because each facet had a specific size, and this size could be directly measured during cutting. In practice, the facets were cut slightly under size, then gradually recut to the appropriate size. This was necessary, as material could always be removed, but if a facet was overcut (too much material removed), the mistake could not be undone. This continued until all facets were in place.

A limitation of such a process is that precise measurements are required for each facet. Yet absolute accuracy is not possible due to human error and the limits of the measuring tool. Consequently, after several facets, errors began to accumulate. These were manifested as facets that did not align properly or that deviated from the prescribed size. When such errors were encountered, the cutting of the preceding facets had to be reevaluated (size and shape) to determine where to recut the inaccurate facet(s). The entire stone was actually cut several times using smaller and smaller iterations until all facets were in place.

The four cavities that held the prongs were cut using a water-cooled dental drill with a superfine burr (3000 grit). Each cavity was placed by measuring the distance from the surrounding facets with a table gauge and then marking the surface with an indelible pen. Depth was gauged visually using a shadow technique with direct reference to the plaster replica.

DISCUSSION

Error Analysis. *Errors from Photography and Scanning.* The photographic method is useful for modeling diamonds to which we have limited access (such as those in various crown jewel collections), where data must be derived from just one or two photographs. This highly iterative method yields a model that is identical to the pictures, but it may still contain errors, as discussed above. As a result, it should only be used when other methods cannot.

Laser scanning is an objective and accurate process that eliminates the assumptions and iterations required with the photographic method for computer modeling. It also has its limitations, however, as evidenced by the number of planar surfaces comprising a single facet on this particular plaster replica, which became obvious when a line drawing was superimposed on its equivalent view from the laser scan (again, see figure 10). This demonstrated that the facets were not optically flat, but rather slightly rounded. This might have been due to the simple cutting equipment used to cut and polish the diamond centuries ago. It could also result from imperfections in the casting process, erosion of the model's surface, or slight deformation of the plaster model with age due to moisture absorption. We addressed this situation by using an average of the angle and index settings for every plane that defined a virtual facet. If the virtual facet's size and shape did not match its counterpart in the line drawing, we changed the angle and/or index (within the range limits defined by the scan planes) until the two matched. Thus, we basically "idealized" the facets to be flat and cut accordingly. However, this process of "iteration of averages" inevitably introduces error (see Error Mitigation below).

Errors Due to the Misaligned Mold for the Plaster Model. Another source of facet error was the fact

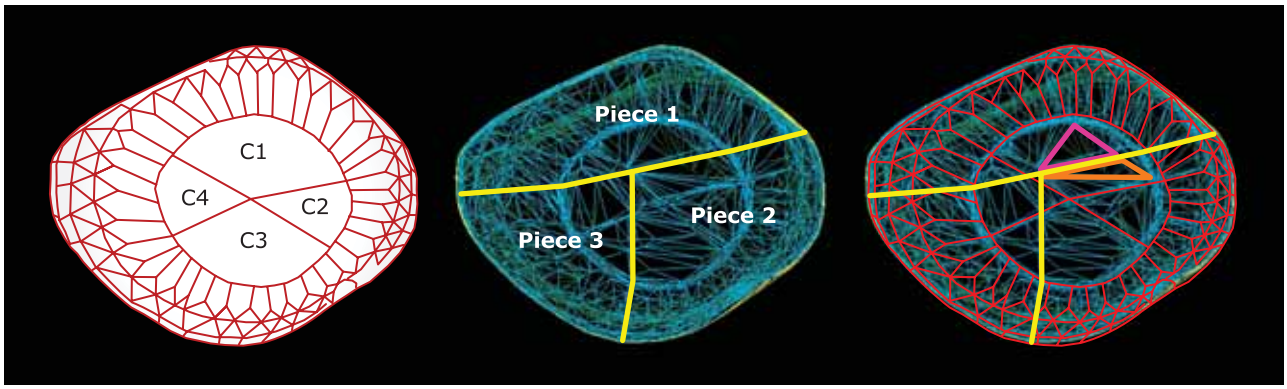


Figure 12. As shown in the line drawing (left), the original Koh-i-Noor had four main crown facets. These were reproduced by the top three pieces of the plaster model, as indicated by the seams shown in the scan data (center). An overlay of the line drawing on the scan data (right) reveals that the two planes defining facet C1 (pink and orange triangles) lie on separate pieces of the model—and that the upper and lower-right pieces of the model were slightly misaligned.

that the plaster replica was created from a mold that came in four sections, one for the bottom and three for the top (figure 12, center). The top three pieces were joined to form a single unit.

Although the seams from the mold pieces were difficult to discern in the scan data, they were clearly visible in the photographs (again, see figure 3). Overlaying the line drawing on the scan image distinctly showed their locations (figure 12, right). Analysis of the facet settings showed that pieces 2 and 3 were in alignment, as the planes on C3 (see figure 12, left) had similar settings regardless of whether the measurement was taken on piece 2 or piece 3. However, measurements on C1 had different settings depending on whether they were taken from pieces 1 and 2, or 1 and 3.

This is because when the model was created, piece 1 was slightly out of alignment, as shown by the angle and index settings of the planes described in figure 12, right: The index (based on a 360 index wheel) and angle settings for the orange portion of facet C1 were 240.21 and 5.57°, respectively, and for the pink portion they were 288.7 and 6.49°. Had piece 1 been properly aligned, these values should have been identical.

The settings for facet C2 were straightforward, as it resided solely on piece 2. For C1, we used the average of the two angles above (~6°) and the lines of intersection with C2 and C4 to calculate an index setting. An index of 250 satisfied all constraints. A similar process was used for C4, as it resides on non-aligned pieces (1 and 3), but the differences in facet settings were negligible.

This misalignment caused the spatial relationships between facets to change, affecting the angle and index settings recorded by the scanner. As a result, these settings could not be used unadjusted when virtually cutting these facets. This limitation was overcome by cutting the facets that overlapped these pieces with reference to the line drawing. Intermediate facets were cut using the relative differences between facet settings.

Error Mitigation. Creation of an accurate model requires accurate line drawings and facet settings. In both cases, however, the process used to maximize accuracy has been shown to introduce error. Additionally, one of the molds used to make the plaster replica was slightly out of alignment, so we had to use assumptions about the facet settings for this portion of the stone. Given these obstacles, how could we generate an accurate computer model?

To this end, we made two assumptions concerning these errors:

1. Any given facet intersection had multiple solutions. For instance, baseline settings for two adjacent facets could yield a satisfactory solution; that is, the virtual facet matched its counterpart on the line drawing. However, on some facets, changing the index by 0.1 and the angle by 0.25° yielded the same intersection, so a *satisfactory* solution might not be the *best* solution.

2. All errors are cumulative. This was borne out when the photographically derived model was compared to the model from the scan data. Although in general there was excellent correlation between the two models and the line drawings, a number of facet settings were considerably different (see figure 13 and table 2).

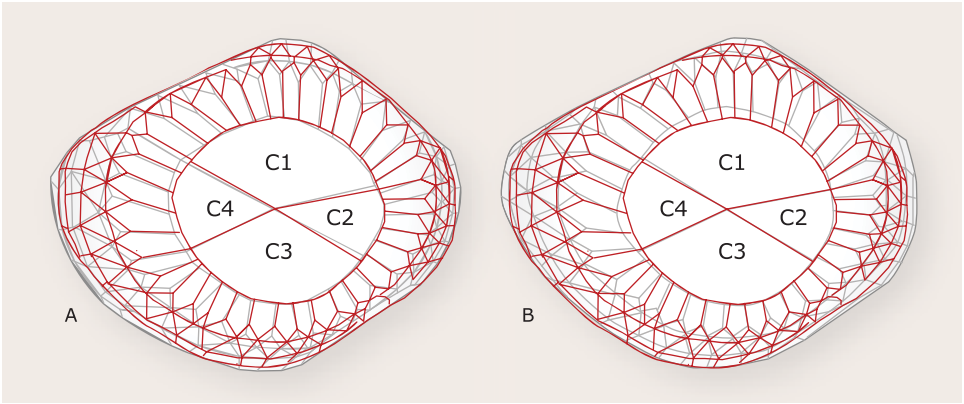


Figure 13. These two images compare angles and index solutions between the computer models (gray) overlaid with the line drawing (red) derived photographically (A) and from the scan data (B).

TABLE 2. Summary of angle and index settings of facets C1–C4 derived from photographic analysis only and using the scan data.

Crown facet	Photo data		Scan data	
	Angle	Index[a]	Angle	Index[a]
C1	9°	195	6°	249.9
C2	10°	325.5	12.5°	317.1
C3	14°	351	17°	347.1
C4	3°	72	7°	5.1

[a] *360 index wheel.*

To compensate for these errors, the "finished" GemCad model was divided into six 60° sectors for a comparison of sidewall angle with the laser-generated model. During computer modeling, eight side views were used and the sidewalls matched. Although there could have been errors in views intermediate to those used, division into six sectors was considered sufficient to verify the accuracy of those intermediate views.

The computer model's outline from each of the six side views was compared to its counterpart view in the scan data (figure 14). Two of the sectors deviated from the correct sidewall angle by around 3°, although their facet patterns matched the line drawings. This was corrected by changing individual facet settings (within the range of settings of the planar surfaces dictated in the scan data) until the sidewall angles matched. The computer model was considered accurate once the sidewall angles matched the scan data and the facet pattern matched all line drawings.

Accuracy can also be corroborated by the fixed rules of geometry that dictate how planar surfaces intersect:

1. The facet pattern on any diamond is a combination of planar intersections (facets), with each plane described by an angle and index setting, and a specified depth of cut. Change any variable for two adjacent facets and their shared edges change, which changes those for their neighbors, and so on throughout the diamond.

2. The irregular, curved surface of the Koh-i-Noor was more difficult to model than a more traditionally cut, symmetrical gem. However, the very complexity of the diamond helped ensure accuracy, as there are a limited number of combinations of 169 facets that can be used to describe a three-dimensional surface as complex as with this shape. The three cutting variables (depth of cut as well as angle and index) result in 507 dependent variables (169 × 3) that define the surface. Although there are many sources of error, we felt that a computer model satisfying 507 dependent variables within the accuracy of GemCad should be the best solution. (The accuracy of GemCad for this model was 40.22 units/mm, or 0.025 mm resolution. The model was magnified approximately 3× onscreen, so nominal resolution was approximately 0.008 mm. However, the program only measures at four-unit intervals, so the actual resolution was closer to 0.03 mm.)

Weight Analysis. The dimensions obtained by computer modeling were derived from the scan data, but there was a 13.17 ct discrepancy between the weight calculated from the scan data and the diamond's reported weight based on these dimensions. Given the known volume, and a density of 3.522 g/cm^3, the scanning software calculated a diamond weight of 204.20 modern carats. However, multiple historical records report a weight of 186.1 old carats (0.2053 g per carat), or 191.03 modern carats (0.2 g per carat). A reduction in weight due to the cavities was considered, but from direct measurement all

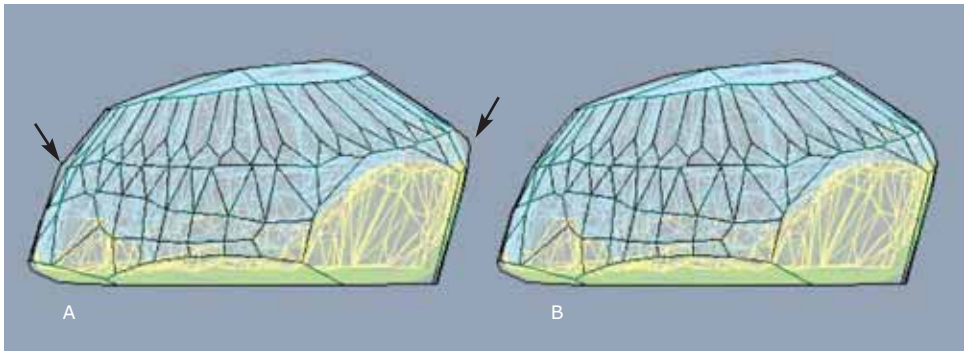


Figure 14. This is a comparison of sidewall angles between the computer model (black) and scan data. In A, the outlines do not correlate (arrows), indicating that errors in the line drawings and computer model exist below the detection threshold. In B, the affected areas have been recut (changing individual facet settings) so the sidewall angles do match.

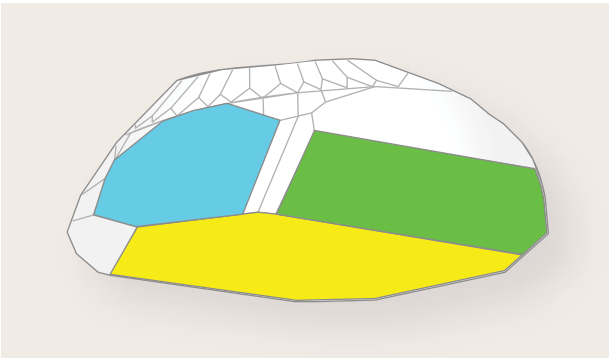
four encompassed an approximate volume equivalent to just 8 points of diamond weight. How could the latest in 21st century technology be at such large variance with a well-documented weight?

The scanned model was made of 155-year-old plaster and had been stored in the damp environment of England. It is possible that the plaster had swollen in the intervening years or during the casting process to attain the volume indicated by the heavier weight. Unfortunately, confirmation of either theory was outside the scope of this experiment. What is known is that a 204.2 ct computer model of the Koh-i-Noor measures 41.78 × 33.63 × 16.56 mm, and if corrected to 191.03 carats (performed through the scanning software and measured in TurboCad), its dimensions become 40.85 × 32.57 × 16.18 mm. This indicates that the plaster expanded 0.97 mm in length (2.3%), 1.06 mm in width (3.3%), and 0.27 mm in depth (2.3%), changing the volume by 7%. Any explanation as to why it would swell more in width than in length or depth would be pure speculation.

Crystallographic Analysis. Once the diamond was modeled on the computer, we used it to test claims by Tennant (1851) and Streeter (1882) that the larger "facets" were cleavages. Tennant studied the original diamond prior to recutting and concluded that the base (figure 15, yellow facet) was a cleavage face and the largest facet (green) was an unpolished cleavage (both being {111}, or octahedral crystal faces). We tested this by merging our computer model with the model of a perfect diamond octahedron featuring small dodecahedral crystal faces. Rotating the Koh-i-Noor within the octahedron confirmed that both the yellow and the green unpolished facets were indeed aligned with {111} faces. The computer model also showed that the blue facet was very close to the index of another octahedral face, but the angle differed by approximately 17°. When the green (unpolished) facet is assumed to be a cleavage (i.e., index 0.0 and angle 0.0°), calculations yield index and angle results of 0.1 and 1.52° for the base (yellow) and 0.4 and 16.69° for the blue facet, respectively. Perfect alignment between a cleavage and a polished facet is not to be expected, as diamond is hardest parallel to a cleavage plane. Grinding or polishing parallel to a cleavage is impossible. If a facet needed to be placed in this approximate configuration, it would have to be offset from perfect alignment.

Ball (1925, p. 342) and Streeter (1882, page 83) speculated that the green unpolished facet was the result of intentional cleaving to damage the diamond prior to handing it over to the British or during some earlier forcible transfer of ownership. The plaster replica and scan data do not support this, as this

Figure 15. This study also investigated historical claims that some of the Koh-i-Noor's original facets were actually cleavages: The area shaded in green, an unpolished cleavage; the base (yellow), a polished face very close to a cleavage; and the blue facet, aligned with the index of a cleavage. When the computer model is merged with the model of a perfect diamond octahedron, the alignment of the green "facet" with one of the cleavage planes suggests that it is an unpolished cleavage plane and the blue facet is close to another cleavage plane, but the angle differs by about 17°.



facet has significantly rounded edges. If the damage was from cleavage performed after the diamond had been polished, someone would have had to regrind the edges to round them and place facets on these surfaces, a somewhat arduous task. Also, the facets surrounding this plane suggest the cutter "blended" them into a preexisting surface; he did not end them abruptly as one would expect if any face were cleaved later. Blending the facets would also be expected to give the diamond a finished appearance while conserving weight. Because of these factors, and a lack of evidence to the contrary, this facet is more than likely the result of natural processes (and subsequent working) rather than intentional damage. Thus, it seems that no damage was inflicted after the Koh-i-Noor was originally cut unless it was disguised by further cutting prior to receipt of the diamond by the British.

This does not explain the unpolished nature of the green facet as described by Tennant. The model indicates it is an octahedral face (111), which would make it physically impossible to polish (although changing the grinding angle by just a few degrees could have permitted this). Perhaps it was a clean cleavage, shiny enough that the cutter felt it needed no further working. Unfortunately, any explanation of why this one facet remained unpolished would be purely conjecture.

Relationship to the Great Mogul Diamond. Ball and Streeter also theorized that the cleavage facets were the result of several diamonds, including the Koh-i-Noor, having been cut from a much larger diamond called the Great Mogul (Ball, 1925, p. 343). According to Tavernier (1676, p. 97), the Great Mogul weighed 279 9/16 ct (286.97 modern ct) and was shaped like half an egg (figure 16). This is the only known historical reference to this diamond's existence. Unfortunately, this diamond subsequently disappeared, negating any truly definitive study.

Streeter and Ball went on to suggest that the Great Mogul disappeared when the Koh-i-Noor was cut from it (figure 17). We tested this hypothesis by comparing computer models of the two diamonds.

The Great Mogul was computer-modeled by superimposing the computer workspace over Tavernier's line drawing of it, similar to the modeling already described. Since Tavernier described the stone as half an egg, a circular cross-section was assumed. We used a photogrammetric process to simultaneously model the diamond in two views, top down (to maintain the circular shape) and from the side (to match the shape and sidewall angles shown in the drawing.) An analysis of the drawing showed changes in the symmetry of each row of facets, but the pattern appeared to be trapezoidal facets alternating with triangles. Different symmetries were tested to see which could generate this facet pattern and still create sidewall angles to match the drawing. A 32-fold symmetry satisfied

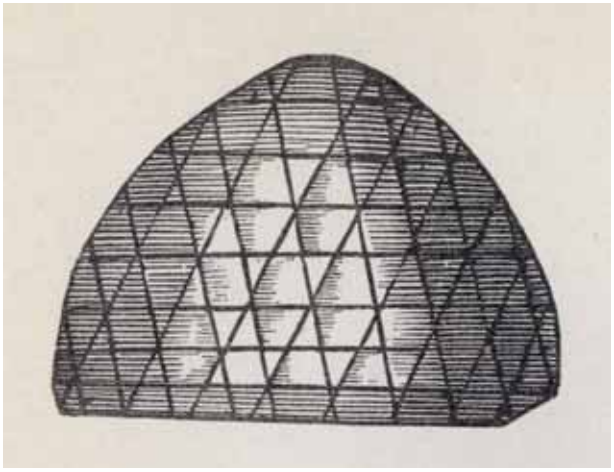
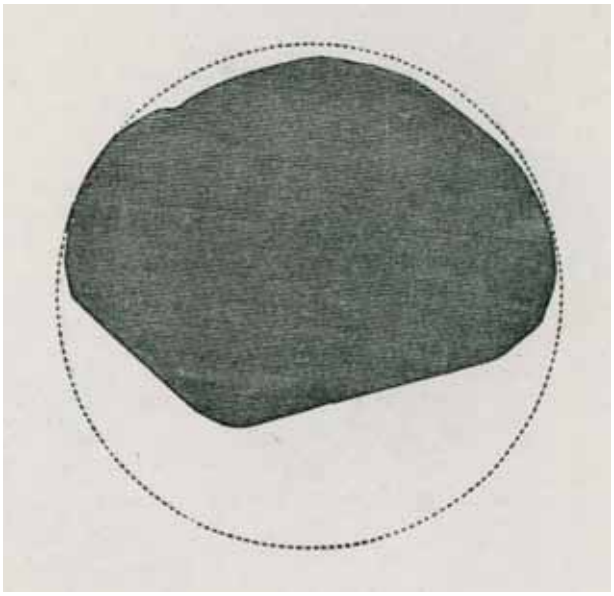


Figure 16. This 17th century drawing published by French gem trader Jean Baptiste Tavernier (1676) is the only known illustration of the Great Mogul diamond.

Figure 17. Streeter (1882) and Ball (1925) hypothesized that the Koh-i-Noor (black area) would have fit inside the Great Mogul diamond (circle). From Ball (1925), p. 342.



these constraints and was used for modeling. Once the computer model was complete, it was sized to encapsulate a volume equivalent to Tavernier's reported weight. This resulted in a diamond 34.85 mm in diameter and 25.61 mm in depth. The Koh-i-Noor was 40.85 × 32.57 × 16.18 mm. The result of superimposing one model on the other is shown in figure 18. Unless Tavernier's reported weight was too low by a large degree, it is clear that there can be no connection between the Great Mogul and the Koh-i-Noor as suggested by Streeter and Ball.

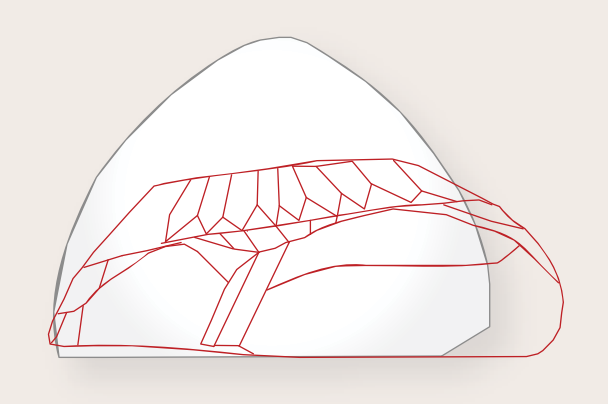


Figure 18. Computer models showed that the Koh-i-Noor (red) could not have been cut from the Great Mogul (black).

Recutting of the Historical Koh-i-Noor to the Modern Diamond. The orientation of the modern Koh-i-Noor within the original version was performed solely as an academic exercise. The modern stone was modeled with a photo from the top-down view and the published dimensions of 36.00 × 31.90 × 13.04 mm. Facet angles were determined by successively iterating pavilion and crown main facets to meet the reported weight of 105.60 ct (Bari, 2001, p. 178).

Once the computer models of both diamonds were merged in GemCad, the orientation of the historical Koh-i-Noor was maintained while the modern version was rotated through various configurations. The number of possible configurations was limited, as the modern diamond was asymmetrically cut. From a top-down view, there are four areas of the girdle that are much flatter than a perfect oval would dictate. These correspond to similar surfaces

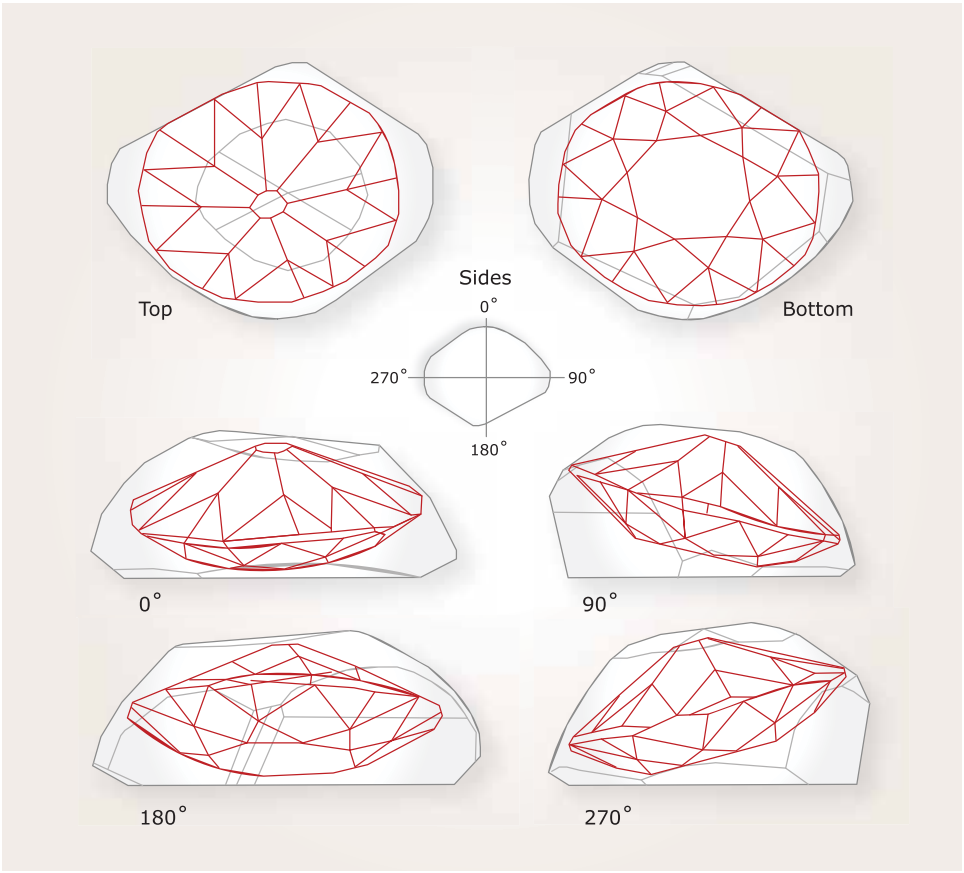


Figure 19. These six illustrations show the most likely orientation of the modern Koh-i-Noor inside the original cut, according to computer modeling. Tennant (1852) stated that it was necessary to turn the stone on its side to remove a flaw where the yellow and the green facets in figure 15 intersect, and to accommodate the removal of an inclusion and fracture in creating the future girdle.



Figure 20. This photo of the CZ replica demonstrates the optical effects that may have been displayed by the original Koh-i-Noor. A 6.5 mm CZ equivalent to a 1 ct diamond is shown for scale. Photo by S. Sucher.

on the historic diamond. Once these were aligned, orientation was a function of tilting the modern model to determine the best fit. Figure 19 shows the approximate orientation of the modern Koh-i-Noor to its original version.

The modern Koh-i-Noor has been described as asymmetrical and lacking brilliance—not an improvement over the original (Streeter, 1882; Balfour, 2000). However, an analysis of the two computer models showed the cutter's limitations. It is obvious that the cutter attempted to create a stone as large and symmetrical as possible, with the former constraint the more important one. Tennant (1852) mentioned no less than five flaws that needed to be removed, imposing additional cutting constraints. Rotating the modern model within the historical model shows that the final orientation removed the existing flaws and maximized both length and width—a larger stone could not have been cut. This also explains the flat spots on the girdle: They were retained to conserve weight, as the cutter knew they would not be quite so apparent when the stone was set.

The lack of brilliance can be attributed to the rather shallow angles on the pavilion main facets, in the vicinity of 27°, about 2.5° above the critical angle of 24.4° for diamond. Crown main angles range from 24–30°. These are a function of girdle placement within the historical stone and the properties dictated by its shape.

Tolkowsky (1919) suggested angles of 40.75° for the pavilion, and 34.5° for the crown for maximum brilliance for a round brilliant cut. Although these values can vary somewhat depending on other factors (size of the table, presence of a culet facet, the facet pattern used), those used on the modern Koh-i-Noor still vary significantly from those that would produce a diamond of, in Tolkowsky's words, "magnificent brilliancy." Unfortunately, the lower angles were necessary to maintain maximum apparent size from a face-up view. Although the stone is not as brilliant or symmetrical as it could be, modeling indicates that the cutter maximized the yield given all known constraints. This was indeed the typical standard for cutting in those days—diamond cutters could be penalized in their pay for not retaining the maximum weight, and beauty was rarely a consideration (Dieulafait, 1874.)

As the CZ replica suggests, the original Koh-i-Noor diamond was brilliant and a unique cut (figure 20). The curved surface seems to magnify reflection, refraction, and dispersion. If this model displays the original optical properties, then it is unfortunate that this diamond was recut.

The replica was subsequently donated to the Natural History Museum in London and is currently on display in their "The Vault" exhibit.

SUMMARY

The photographic and laser scan analysis of the plaster model of the historic Koh-i-Noor diamond shows that the original diamond had 169 facets, with

dimensions of 40.85 × 32.57 × 16.18 mm. It had a high-domed crown, flat base, and predominantly triangular and rectangular facets covering its surface, reminiscent of other Mogul-cut diamonds in the Iranian Crown Jewels. Although the scan data suggested a 204.20 ct stone, rather than the reported 191.03 ct, the authors surmise that this was due to expansion of the plaster model by 7%, either as a result of water absorption over the past 150+ years or as an artifact of the casting process.

A crystallographic analysis confirmed earlier reports that the base and largest facet are aligned with cleavage faces. A third facet was close to the index of another cleavage face, but deviated by 17°. This information also was used to determine that no intentional damage was inflicted on the original stone during any previous transfer of ownership, as suggested by earlier reports. It also showed that the Koh-i-Noor could not have been cut from the Great Mogul diamond, as the latter was too small, discounting another theory.

An analysis of the recutting of the original Koh-i-Noor to its modern version showed that the cutter was operating under tight constraints. Maximum apparent size was paramount, but this came at the expense of symmetry and brilliance. However, the cutter did a superlative job within the constraints, as a stone of larger dimensions would not have been physically possible.

Creation of a cubic zirconia replica offers the best approximation of the brilliance and optics of the original stone to date. It is the opinion of the authors that this was a magnificent example of ancient cutting technique, with optics unusual for a faceted diamond, and that something of historic value was lost when it was recut.

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